Epidemics in Networks
Part 3 — Qualitative observations of disease spread in networks

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Recall our key questions

For SIR:

- $\mathcal{P}$, the probability of an epidemic.
- $\mathcal{A}$, the "attack rate": the fraction infected if an epidemic happens (better named the attack ratio).
- $\mathcal{R}_0$, the average number of infections caused by those infected early in the epidemic.
- $I(t)$, the time course of the epidemic.

For SIS:

- $\mathcal{P}$
- $I(\infty)$, the equilibrium level of infection
- $\mathcal{R}_0$
- $I(t)$

Assumptions

We start with some simple assumptions:

- SIS or SIR disease on a fixed static network.
- Susceptible nodes , infected nodes , and recovered nodes .

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- SIS or SIR disease on a fixed static network.
- Susceptible nodes , infected nodes , and recovered nodes .
- Disease transmits along an edge with rate β (many authors use τ)
- Infected individuals recover with rate γ

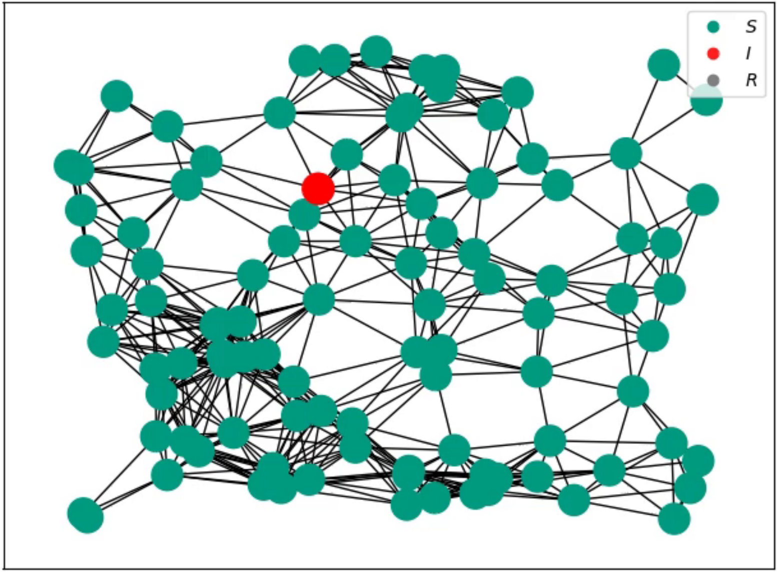
Introduction

Sample stochastic simulations

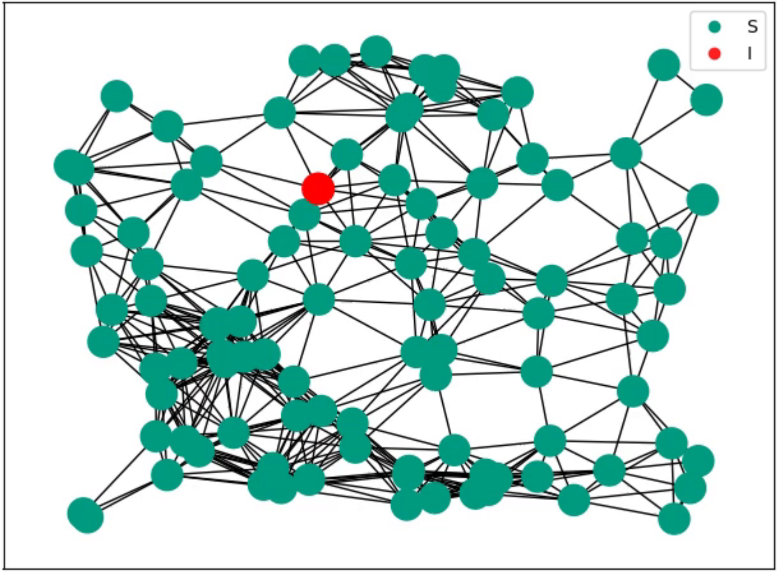
Impact of network properties

References

Sample SIR epidemic



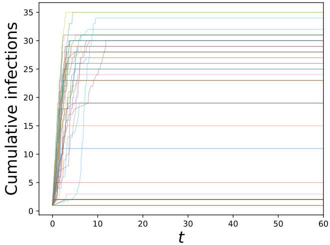
Sample SIS epidemic



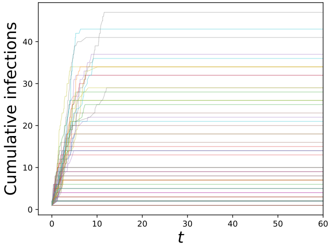
Stochastic simulation — SIR on network case

SIR disease spread with $\langle K \rangle = 5$, $\beta = 0.4$, and $\gamma = 1$.

$P(5) = 1$,



$P(1) = P(9) = 0.5$



$N = 50$

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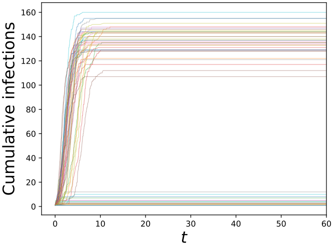

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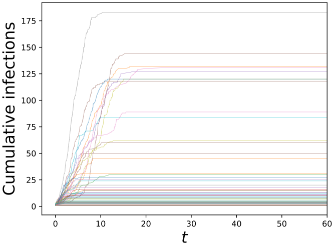

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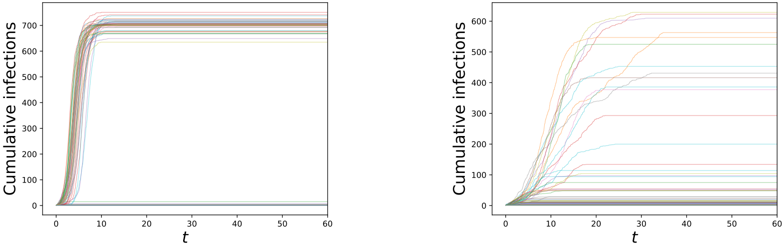

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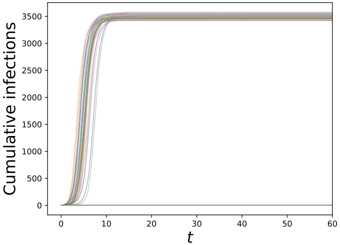

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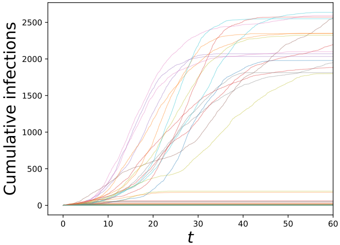

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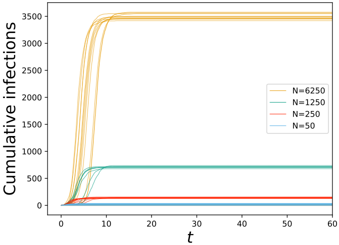

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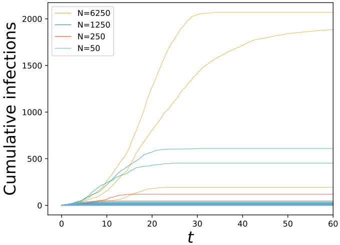

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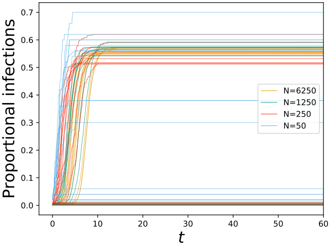

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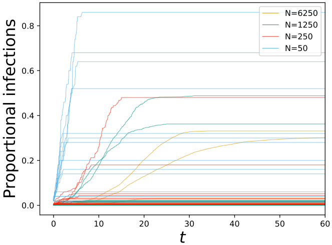

SIR observations

- In large networks outbreaks are either small (non-epidemic) or large (epidemic).
- Small outbreaks don't care about network size (once network is sufficiently large).
- Epidemic sizes are proportional to network size.
- The degree distribution affects the final size and the early growth.

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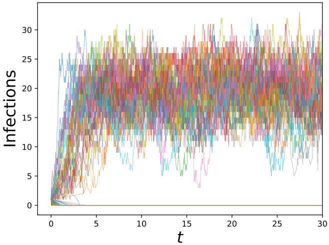

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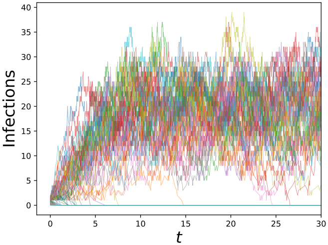

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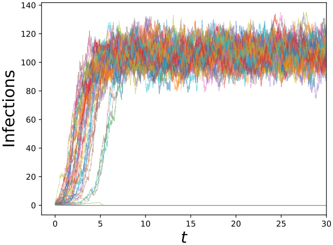

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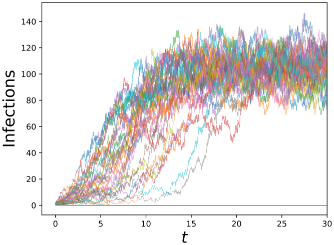

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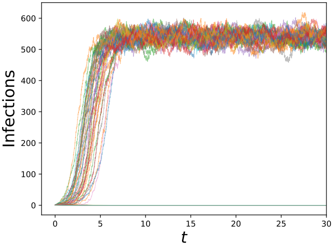

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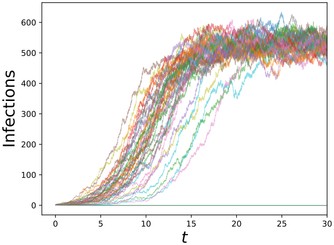

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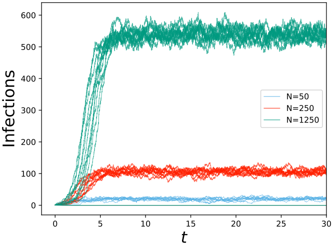

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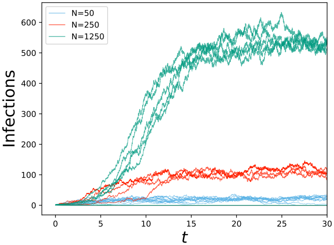

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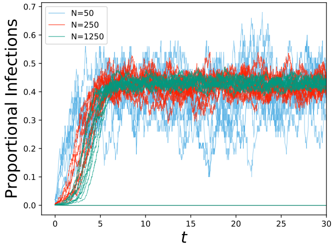

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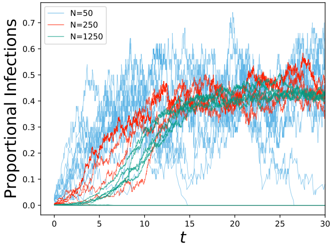

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What does the "equilibrium" distribution look like?

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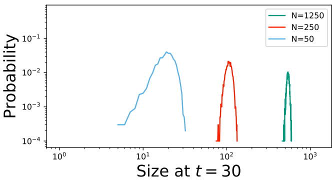

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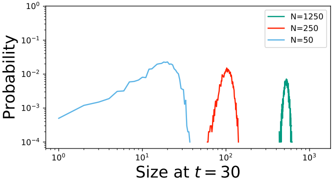

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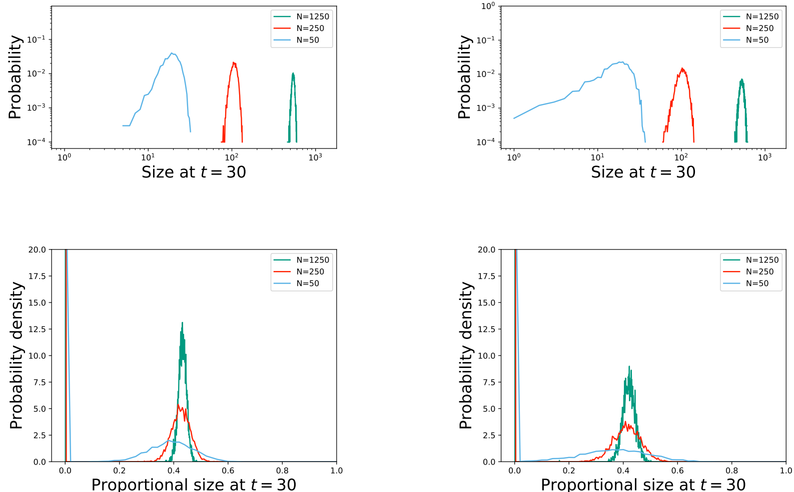
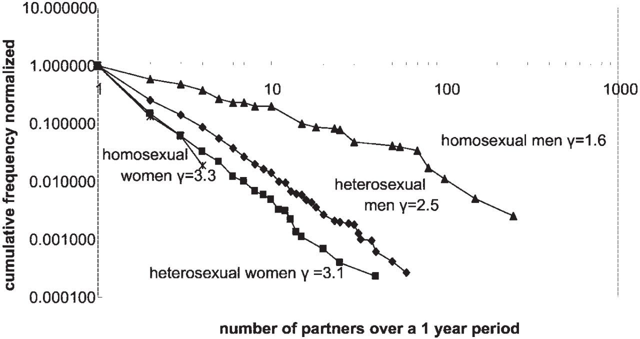


SIS observations

- In large networks outbreaks either go extinct quickly (non-epidemic) or reach an endemic equilibrium (epidemic).
- Small outbreaks don't care about network size.
- Epidemic equilibrium sizes are proportional to network size.
- Coefficient of variation decreases for large networks. [typical deviation from mean is small compared to mean.]

Degree distribution

From [1]:



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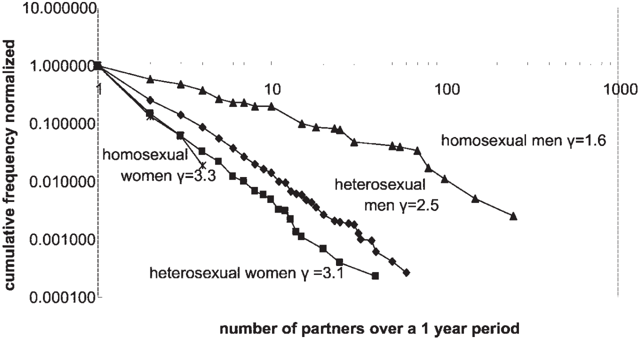

Impact on $\mathcal{R}_0$:

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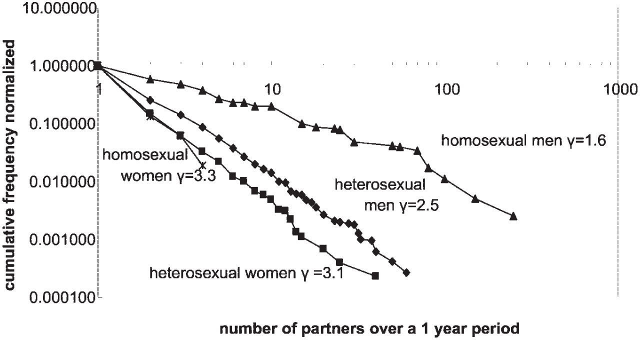

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Holding $\langle K \rangle$ fixed, increasing heterogeneity increases $\mathcal{R}_0$.

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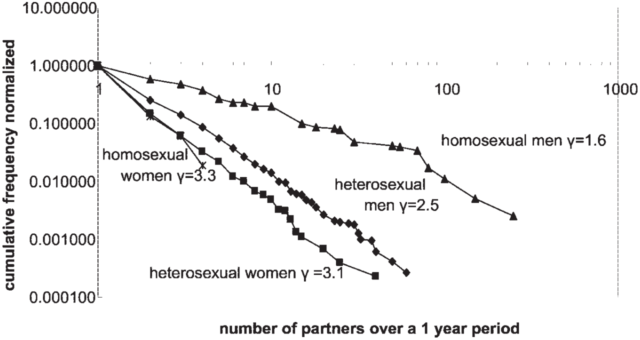

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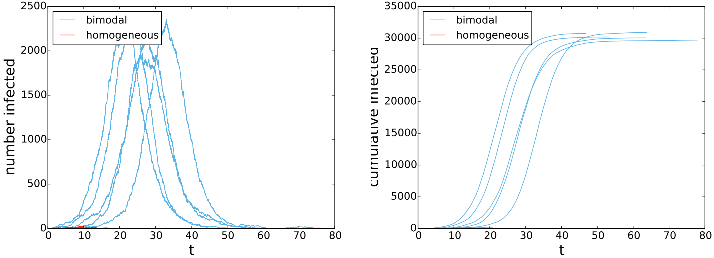
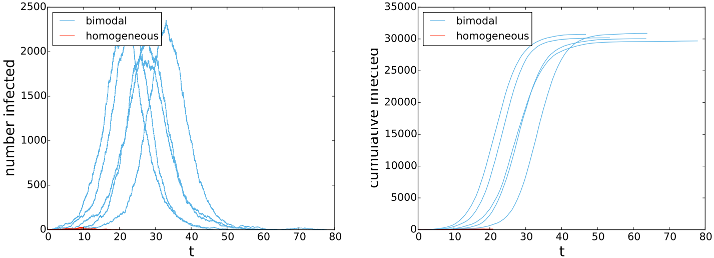
Why does this increase $\mathcal{R}_0$?

Impact of degree distribution



Small transmission rate

Impact of degree distribution



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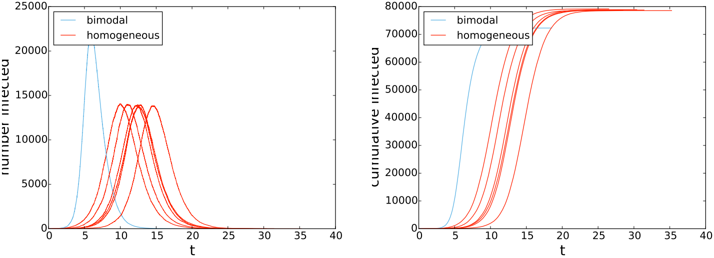

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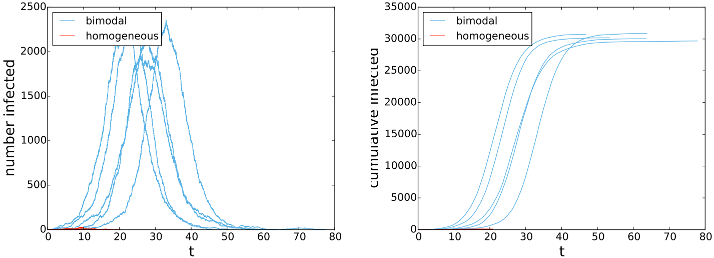

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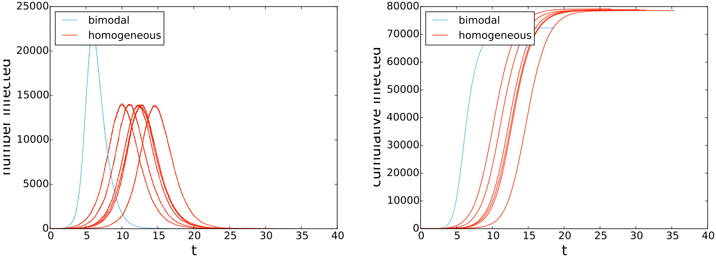

Large transmission rate

Understanding impact on final size

- Why does degree heterogeneity increase final epidemic size at smaller transmission rate?

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- Why does degree heterogeneity decrease final epidemic size at higher transmission rate?

Degree correlations

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People claim opposites attract, but really they tend not to. Individuals likely form partnerships with similar individuals.

If high degree individuals preferentially contact high degree individuals, impact on $\mathcal{R}_0$: Increases it.

Degree correlations

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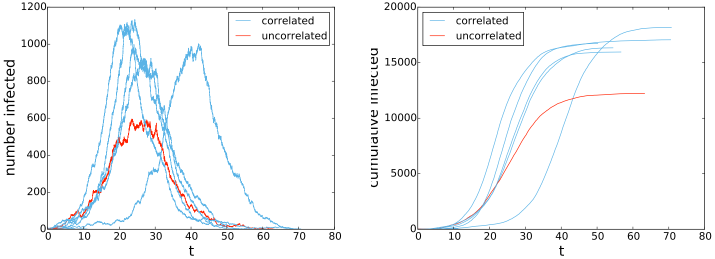
- Why do degree correlations decrease final epidemic size at higher transmission rate?

Degree correlations



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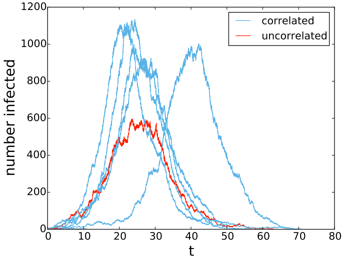

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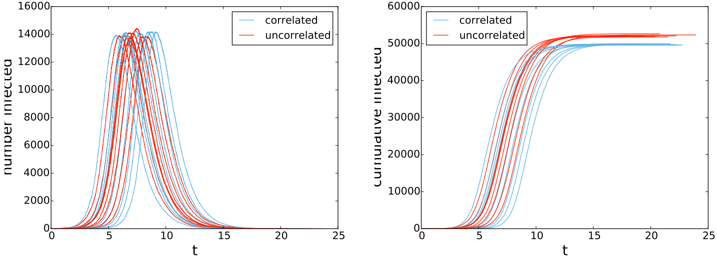

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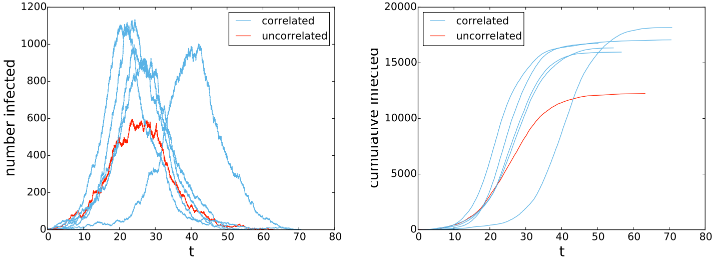

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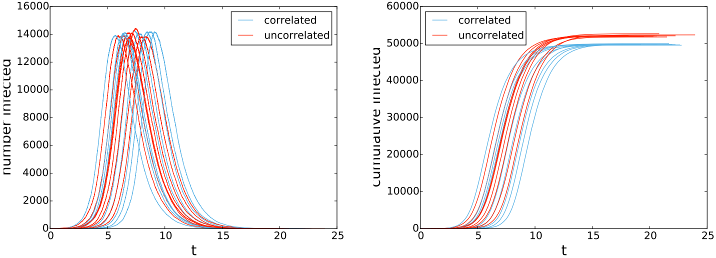

Large transmission rate

Partnership duration

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For SIR, long partnership duration decreases $\mathcal{R}_0$ because repeated transmissions are wasted.

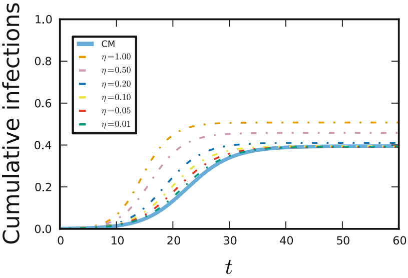
Partnership duration

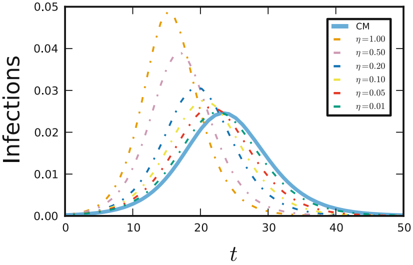
If partnerships have long duration, people are likely to have some transmissions blocked, and are likely to reinfect their infector (in SIS) rather than someone else.

Impact on $\mathcal{R}_0$:
For SIR, long partnership duration decreases $\mathcal{R}_0$ because repeated transmissions are wasted.
For SIS, it is complex — repeated transmissions are wasted, but long-lasting partnerships help ensure that newly-recovered high degree nodes are quickly reinfected [2].

Partnership duration

Sample SIR epidemics from [3]





(η is inverse partnership duration, “CM” is static Configuration Model)

Clustering

If partnerships are clustered, even early on individuals who become infected are likely to have partners who are infected by others.

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Clustering

If partnerships are clustered, even early on individuals who become infected are likely to have partners who are infected by others.

Impact on $\mathcal{R}_0$: For SIR, decreases it. For SIS, it is complex.

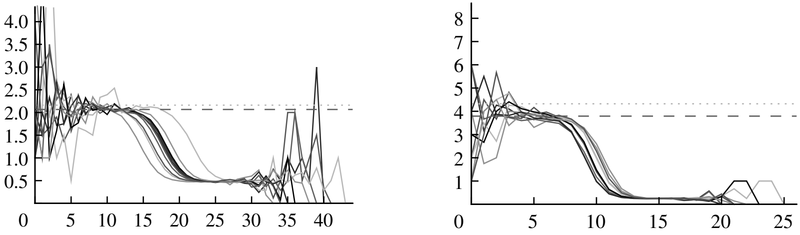
Clustering

Ratio of successive generation sizes from [4]

Clustering

Ratio of successive generation sizes from [4]

lower transmission rate higher transmission rate



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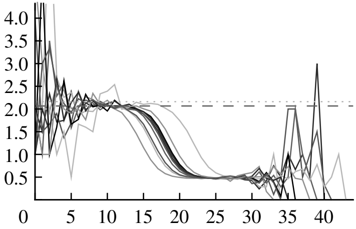
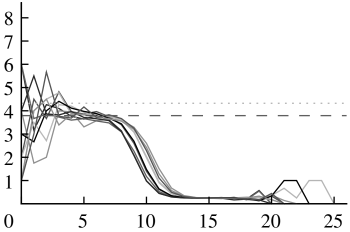


higher transmission rate



Dotted line is prediction ignoring clustering. Dashed line is correction accounting for triangles and squares.

Clustering

But the size is not so affected:

Clustering

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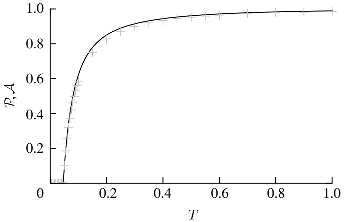
Comparison of unclustered prediction (line) with stochastic simulation (symbols)



(horizontal axis is transmission probability, vertical is fraction infected.)

Introduction

Sample stochastic simulations

Impact of network properties

References

References I

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